CMPU-224 Lab3 Quiz - Solutions

Spring 2025

1. You are given the following 8-bit two's-complement number: `10001000`
   (a) (1 point) What is the decimal value of this number?

   Solution: -120
   The MSB is 1, so it is negative. There is a one in bit-7 and bit-3. Bit-7 has the weight of -128 and bit-3 has the weight of 8, giving $-128 + 8 = -120$.
   Or you can take the two's complement to get the magnitude of the number. Invert the bits: `01110111`, and add 1: `01111000` ($64 + 32 + 16 + 8 = 120$). Result: -120.

   (b) (1 point) What is representation of that number as a 10-bit two's-complement number? Give your answer as in hexadecimal.

   Solution: 0x388
   Sign extend the 8-bit number `1000 1000` to 10 bits by copying the MSB (1) to the new upper bits: `11 1000 1000`. Grouped for Hex: `11 1000 1000` $\rightarrow$ `0x388`.

2. For this question, assume all the numbers are internally represented as 5-bit two's complement binary numbers.
   (a) (1 point) What is the decimal value of $10 + 8$?

   Solution: -14
   $10_{10} = 01010_2$, $8_{10} = 01000_2$.
   $01010 + 01000 = 10010_2$. This is a negative number (MSB=1).
   Invert `01101` + 1 = `01110` (14_{10}). Result: -14.

   (b) (1 point) The above answer is an example of (choose one):
   A. Positive Overflow
   B. Negative Overflow
   C. Modular arithmetic
   D. None of the above

   Solution: A. Positive Overflow
   Adding two positive numbers resulted in a negative number because the result +18 exceeds the max positive value (+15) for 5 bits.

(c) (1 point) What is value of $-9 \gg 1$? Give your answer as a decimal number.

Solution: -5
$-9_{10} = 10111_2$. Arithmetic right shift preserves the sign bit: `11011`.
Invert `00100` + 1 = `00101` (5_{10}). Result: -5.
Or remember that a right shift of one is the same as dividing by two and rounding down. $-9/2 = -4.5 \rightarrow$ rounding down gives -5

3. Consider a 12-bit floating point representation called "TinyFloat". Bits: 11 (Sign), 10-6 (Exp), 5-0 (Frac).

(a) (1 point) In the above TinyFloat system, what is the bias used to calculate exp?

Solution: 15
Bias formula: $2^{k-1} - 1$ where $k = 5$. $2^4 - 1 = 15$.

(b) (1 point) Consider the following Tiny Float number: `0x3DC`. What is the decimal value?

Solution: 1.4375
Binary: `0011 1101 1100`. $S = 0$, $Exp = 15$, $Frac = 011100$.
$E = 15 - 15 = 0$. Significand $= 1.frac = 1.011100_2$.
$1 + 1/4 + 1/8 + 1/16 = 1 + 0.25 + 0.125 + 0.0625 = 1.4375$.

4. (3 points) Convert the decimal number 12.2 into the TinyFloat representation described above. Give the binary results for the sign, exp, and frac fields.

Solution: Sign: 0, Exp: 10010, Frac: 100010

- $12 = 1100_2$. $0.2 \approx .00110011...$
- Combined: $1100.0011... = 1.1000011... \times 2^3$.
- Exp: $3 + 15 = 18 = 10010_2$.
- Frac: We need 6 bits. Significand bits are `100001 10011...`
- Rounding: The extra bits are greater than a half, so we round up: `100001` becomes `100010`.